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# A simple note on Herd Behaviour\*

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Abstract

In his 'Simple model of herd behaviour', Banerjee (1992) shows that – in a sequential game – if the first two players have chosen the same action, player 3 and all subsequent players will ignore his/her own information and start a herd, an irreversible one. In this paper we analyse the role played by the tie-breaking assumptions in reaching the equilibrium. We showed that: players' strategies are parameter dependent; an incorrect herd could be reversed; a correct herd is irreversible.

*Keywords:* Herd behaviour, Run. *JEL classification:* D8

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#### 1. Introduction

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In the last decade, studies on 'herding' were abundant. Herd behaviour refers to the phenomenon according to which people follow the example of other people ignoring their private information. This kind of behaviour was first pointed out by Becker (1991) and was first formally analysed by Banerjee (1992) and Bikhchandeni *et al.* (1992).

Studying herd behaviour could be useful to explain countless social and economic issues. In the real world (usually) people make their decision sequentially. In the process of decision making, agents observe the decisions taken by previous agents, and may be influenced by that information.

The basic idea of herd behaviour is very simple: ignoring private information and joining the queue. The assumption that agents possess rational expectations is usually being used to assert that an agent, although not familiar with the true model, could reach an efficient outcome drawing on all available information including the one acquired observing other people's actions.

Banerjee in his analysis tries to demonstrate the hypothesis of 'herd behaviour': everyone is doing what everyone else is doing, even when his or her private information suggests acting differently. The model is a sequential decision-making game with one wining action and three possible information states of the subjects: correct, wrong or no information. Banerjee shows that as soon as the first two players choose the same action, all subsequent players will follow them, independently of their private information. Such an irreversible queue ("lock-in") – caused by the individual rational behaviour to gain a better information position by looking at what other agents are doing – may result in a Pareto non-optimal equilibrium since the imitated action may be a non-winning one.

Banerjee's model has two important features: it has both a very simple and intuitive structure and it gives very strong results.

Subjects are rational, and sometimes they are indifferent among several possible actions to break this *an pass* a set of tie-breaking rules is needed, indicating what he/she does in any occasion in which he/she is indifferent between two or more actions. Using Banerjee's words, "each of these assumptions is made to minimize the possibility of herding" (Banerjee, 1992: 803).

**Assumption A.** Whenever a decision-maker has no signal and everyone else has chosen zero, he/she always chooses zero.

**Assumption B.** When decision-makers are indifferent between following their own signal or someone else's choice, they always follow their own signal.

Assumption C. When a decision-maker is indifferent between two or more of the choices made by the previous decision-makers, he/she chooses to follow the one which has the highest value.

It can easily be seen that the rules chosen by Banerjee are not the only plausible ones. In what follows, we will therefore examine if there is any critical relation between the tie-breaking rules and the overall dynamics of the model. In particular, we will show that by choosing an alternative decision rule in case that the first player has no signal, the possibility of an irreversible queue starting from the first player is generally smaller.

Developing a model of herd behaviour which draws on Banerjee's model has some *pros* and *cons*. On the one hand, herding models a *la* Banerjee are more general compared to Informational cascades one (see Bikhchandeni *et al.* 1992). For this reason a generalization of it can produce a wide-ranging conclusions<sup>1</sup>. On the other hand, even a minor modification of Banerjee's model would produce a more complex analytical solution.

The main goal of this paper is to present some possible extensions to Banerjee's model, which will explore the robustness of the original results, as well as the presence of analytical complexities in the extended models.

#### 2. "A simple model of herd behaviour" under different assumption sets

Let  $A = [0, 1] \in R$  be the set of all possible investments, where only  $a^* \in A$  pays a positive pay-off. Let  $S = [0, 1] \in R$  be the set of all possible signals, where only  $s^* \in S$  signals to invests in  $a^*$ . The aim of the game is to invest in  $a^*$ . The pay-off is one when action  $a^*$  is chosen and zero otherwise. There is a population of N players who take their decision sequentially and in a fixed order. Each player knows the choices made by those before him/her but is not aware of the information on which these choices were based. Let indicate with  $\alpha$  the probability of receiving a signal and with  $\beta$  the probability that the signal is correct.

The decision-making process is based on the following three tie-breaking rules (we will follow the notation adopted by Banerjee and refer to these rules as assumptions):

**Assumption A1.** Whenever the first decision-maker has no signal, he/she chooses randomly an action from the set of all possible actions.

**Assumption B1.** When decision-makers are indifferent between following their own signal or someone else's choice, he/she chooses randomly an action from the set of all possible actions.

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Assumption C1. When a decision-maker is indifferent between two or more of the choices made by the previous decision-makers, she/he chooses randomly among the actions that leave his/her utility unchanged.

# 2.1 Assumption A vs. Assumption A1

We can demonstrate<sup>2</sup>, however, that the dynamics of the model may change if we replace Assumption A with Assumption A1.

The expected pay-off for the Subject is zero under both assumptions. Under Assumption A, however, an action equal to zero is a clear signal for the subsequent players that the decision-maker did not receive a signal. Thus, the first player in the sequence who chooses an action different from zero must have received a signal. In the case that the subsequent player does not receive a signal he/she will follow the choice of his/her predecessor. Since two identical actions are more likely to be correct than incorrect, the next player will follow the queue even if he/she receives a signal suggesting acting differently, and consequently, all the next players will follow this action. Under Assumption A1 however, the first player will choose an action different from zero in case he/she has not received a signal. Thus, a queue starting from the first player is less informative for the next decision-makers than under Assumption A.

#### 2.2 Assumption B vs. Assumption B1

Note that replacing Assumption B with assumption B1 does not affect the results of the game, in fact whatever subject *i* will play a herd will start.

#### 2.3 Assumption C vs. Assumption C1

Note that, compared to Assumption C, the information that can be deduced out of an observed sequence of actions would now be generally less precise. Under Assumption C, two non-consequent actions, which are not the option with the highest value, have to be based on a correct signal (since the probability that two wrong signals match is zero, and since in the case of no-signal the option with the highest value is chosen). However, under Assumption C1, two non-consequent actions may be either both correct or both incorrect. Two scenarios are plausible: in the first scenario both players receive the same signal, which means that it is the wining one and, thus, the action is

<sup>&</sup>lt;sup>1</sup> For an application of Bikhchandeni et al. (1992) model in a market contest see Hey and Morone (2004), and Morone (2005). <sup>2</sup> For a proof see Morone (1999), and Morone and Samanidou (2006)

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correct; in the second scenario only the first among the two players receive a signal while the second chooses, randomly, to follow the action of the first player (in this case the action chosen will be correct only if the signal of the first player was the correct one).

Though under Assumption C1 the sequence of actions observed by the decision-makers may be less informative than under Assumption C, the overall decision of the model is not affected. Note that under Assumption C1 the probability that two non-consequent identical actions are correct is smaller than under Assumption C, nonetheless, it is still higher than the probability that two non-consequent identical actions are wrong. Thus, the subsequent agents who do not receive a signal or do receive a signal which, does not match with the options chosen by previous players, will decide to follow (as they would have done anyway under Assumption C).

#### 3. The simulation

For different combinations of  $\alpha$  and  $\beta$  we simulated 1000 runs with 100 agents per run for Set 1 (i.e. the original model of Banerjee) and Set 2 (i.e. the revised model under Assumption A1). The simulations were run in GAUSS. For each agent we drew three numbers from a uniform random distribution between 0 and 1. If the first number drawn was less than or equal to  $\alpha$ , we assumed that the player had received a signal. This signal was the correct one if the second number drawn was less than or equal to  $\beta$ , and wrong otherwise. Finally, the third number drawn represented the wrong signal. We used the same random numbers for Set 1 and Set 2, so that different results may emerge solely due to the variation of Assumption A from Assumption A1.

# 4. Results

Figure 1 shows the probability of herding for different  $\alpha$  and  $\beta$  in both sets. More precisely, it illustrates the proportion of 'herd decisions' out of the total number of decisions taken (i.e. 100\*1000 decisions per each combination of  $\alpha$  and  $\beta$ ). 'Herd decisions' are those taken in the case where the player did get a signal but chose to ignore it. Even though for each combination the probability of herding is nearly the same for Set 1 and Set 2, the probability in the case of Set 2 will always be smaller.

In Figure 2 and 3, on the other hand, we report the probability of observing irreversible runs. In other words, the proportion of irreversible runs out of the total number of 1000 runs in each combination of the parameters. In the case of Set 1, a run is irreversible and correct if at least two subjects played the correct action. A run is irreversible and incorrect if at least two subjects played

the same incorrect action and no correct action was played before. In the case of Set 2, a run is irreversible and correct in two cases: first, when the correct action is played by all the first players and the length of the *critical queue* is reached; secondly, when at least two subjects played the correct action, which has been played at least once before the length of the *critical queue* was reached. Conversely, a run is irreversible and incorrect in two cases: first, when all the players choose the same incorrect action and the length of the *critical queue* is reached; secondly, when at least two subjects played the same incorrect action and no correct action has occurred before. Table 1 shows that under Assumption A1 the probability of a correct irreversible run will therefore be always smaller under Assumption A1 than under Assumption A. It is worth noting that, by using 100 agents, we reduced the probability of observing a reversible run under both assumptions to nearly zero, for all the parameter sets we considered. Under Assumption A1, on the other hand, this scenario is delayed by one lag. This delay allows for the aggregation of some additional information, so discovering the correct action is more likely.



It is evident that irreversible herding entails some very undesirable features. In fact, when an irreversible queue starts, agents are trapped in it and cannot do anything else but following. For this reason, it would be appropriate to study the composition of irreversible herding. As can be seen in

Figure 4, the proportion of correct actions is generally bigger under Assumption A1 (Set 2) than under Assumption A (Set 1). Set 2 is therefore more efficient than Set 1, or in other words, Set 1 is Pareto-dominated by Set 2.





Figure 6

Figure 5 and 6 demonstrates that the ratio of incorrect signals to correct actions is always higher in Set 2 than in Set 1, while the ratio of correct signals to incorrect actions is always lower in Set 2. In consequence, the proportion of incorrect signals being 'wasted' in favour of the correct action is bigger in Set 2, whereas the proportion of correct signals being 'wasted' in favour of an incorrect action is smaller in Set 2.

## 5. Conclusion

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In this work we extended Banerjee's model of herd behaviour replacing one of its fundamental and 'innocuous' assumptions. More precisely, we replaced Assumption A (whenever a decision-maker has no signal and everyone else has chosen zero, he/she will always choose zero), with Assumption A1 (whenever a decision-maker has no signal and everyone else has chosen zero, he/she will always choose randomly among all possible actions).

The consequence of this slight change in the assumptions' set leads to an important alteration in the players' strategy. On the one hand, in Banerjee's model two identical actions are enough to generate an irreversible queue. This fact results in a loss of information with the consequence of a non optimal aggregated result. On the other hand, in the modified model players' strategies are parameter dependent: private information will not be systematically ignored in the presence of a queue. Our results are extremely important if compared to various Bikhchandeni's generalizations. Since we can conclude that changes in the apparently innocuous tie-breaking rules produces a change in agents strategy.

To corroborate our findings, we developed a simulation programme able to replicate the subjects' behaviour under both assumption sets. Although the simulation results showed rather small differences between the two models (e.g. the probability of herding differed by

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approximately 5%), the main result of our study is given by the fundamental change in the agents' strategies. Furthermore, we believe that another important improvement that our model has offered to Banerjee's findings is the introduction of the possibility of agents to decide not to join the crowd at an early stage of the cascade.

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